Math 53, Discussions 116 and 118 **2022-10-07**

More tangent planes, and some review

Questions

Problem 1. Last time, I asked you to find the tangent plane to the surface

$$xy + yz + zx = 5$$

at the point $(1, 2, 1)$. Do it again, but using the method you learned yesterday in lecture.

Problem 2. Let S be the cone $x^2 + y^2 = z^2$ and let H be the plane $x - 2y + 3z = 13$. The curve of intersection $C = S \cap H$ is an ellipse, and the point $P(4, 3, 5)$ is on this ellipse.

(a) Find the tangent plane to S at the point P.
(b) The plane from (a) and the plane H intersect in a line. Parametrize this line.
(c) Find the two possible unit tangents $\mathbf{T}$ to the curve C at the point P.
(d) Find the unit normal $\mathbf{N}$ to the curve C at the point P. This one is conceptually tricky. Here are some observations to help you. The curve C is contained in the plane H, so $\mathbf{N}$ must be parallel to this plane. Also, $\mathbf{N}$ is orthogonal to $\mathbf{T}$, and it points in the direction the curve is "turning."